



INTERNATIONAL STANDARD ISO 4185:1980  
TECHNICAL CORRIGENDUM 1

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INTERNATIONAL ORGANIZATION FOR STANDARDIZATION • МЕЖДУНАРОДНАЯ ОРГАНИЗАЦИЯ ПО СТАНДАРТИЗАЦИИ • ORGANISATION INTERNATIONALE DE NORMALISATION

# Measurement of liquid flow in closed conduits — Weighing method

## TECHNICAL CORRIGENDUM 1

*Mesure de débit des liquides dans les conduites fermées — Méthode par pesée*

*RECTIFICATIF TECHNIQUE 1*

Technical corrigendum 1 to International Standard ISO 4185:1980 was prepared by Technical Committee ISO/TC 30, *Measurement of fluid flow in closed conduits*.

Page 2

### 1.5 Notation

Designation of  $\rho_a$ , delete "(at 20 °C and 1 bar\*)"; delete the footnote "\* 1 bar = 10<sup>5</sup> Pa".

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Annex C, equation (1)

Replace the existing equation (1) with the following equation (1). (The brackets have been enlarged to include the entire fraction.)

$$s_x = \left[ \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1} \right]^{1/2} \quad \dots(1)$$

UDC 532.575:531.753

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**Descriptors:** flow measurement, liquid flow, pipe flow, measuring instruments, flowmeters, calibration, weight measurement, error analysis.

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# International Standard



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INTERNATIONAL ORGANIZATION FOR STANDARDIZATION • МЕЖДУНАРОДНАЯ ОРГАНИЗАЦИЯ ПО СТАНДАРТИЗАЦИИ • ORGANISATION INTERNATIONALE DE NORMALISATION

## Measurement of liquid flow in closed conduits — Weighing method

*Mesure de débit des liquides dans les conduites fermées — Méthode par pesée*

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4185-1980 (E)

## Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards institutes (ISO member bodies). The work of developing International Standards is carried out through ISO technical committees. Every member body interested in a subject for which a technical committee has been set up has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work.

Draft International Standards adopted by the technical committees are circulated to the member bodies for approval before their acceptance as International Standards by the ISO Council.

International Standard ISO 4185 was developed by Technical Committee ISO/TC 30, *Measurement of fluid flow in closed conduits*, and was circulated to the member bodies in August 1978.

It has been approved by the member bodies of the following countries :

Australia	Germany, F.R.	Poland
Belgium	India	Romania
Brazil	Italy	Spain
Chile	Korea, Rep. of	United Kingdom
Czechoslovakia	Mexico	USA
Egypt, Arab Rep. of	Netherlands	USSR
France	Norway	Yugoslavia

The member bodies of the following countries expressed disapproval of the document on technical grounds :

Japan  
South Africa, Rep. of

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# Measurement of liquid flow in closed conduits — Weighing method

## 1 General

### 1.1 Scope and field of application

This International Standard specifies a method of liquid flow-rate measurement in closed conduits by measuring the mass of liquid delivered into a weighing tank in a known time interval. It deals in particular with the measuring apparatus, the procedure, the method for calculating the flow-rate and the uncertainties associated with the measurement.

The method described may be applied to any liquid provided that its vapour pressure is such that any escape of liquid from the weighing tank by vaporization is not sufficient to affect the required measurement accuracy. Closed weighing tanks and their application to the flow measurement of liquids of high vapour pressure are not considered in this International Standard.

This International Standard does not cover the cases of corrosive or toxic liquids.

Theoretically, there is no limit to the application of this method which is used generally in fixed laboratory installations only. However, for economic reasons, usual hydraulic laboratories using this method can produce flow-rates of 1.5 m<sup>3</sup>/s or less.

Owing to its high potential accuracy, this method is often used as a primary method for calibration of other methods or devices for mass flow-rate measurement or volume flow-rate measurement provided that the density of the liquid is known accurately. It must be ensured that the pipeline is running full with no air or vapour pockets present in the measuring section.

### 1.2 References

ISO 4006, *Measurement of fluid flow in closed conduits — Vocabulary and symbols.*

ISO 5168, *Measurement of fluid flow — Estimation of uncertainty of a flow-rate measurement.*

OIML, Recommendations Nos. 1, 2, 3, 20, 28, 33.

### 1.3 Definitions

Only terms which are used in a special sense or the meaning of which merits restatement are defined below.

**1.3.1 static weighing :** The method in which the net mass of liquid collected is deduced from tare and gross weighings made respectively before and after the liquid has been diverted for a measured time interval into the weighing tank.

**1.3.2 dynamic weighing :** The method in which the net mass of liquid collected is deduced from weighings made while fluid flow is being delivered into the weighing tank. (A diverter is not required with this method.)

**1.3.3 diverter :** A device which diverts the flow either to the weighing tank or to its by-pass without changing the flow-rate during the measurement interval.

**1.3.4 flow stabilizer :** A structure forming part of the measuring system, ensuring a stable flow-rate in the conduit being supplied with liquid; for example, a constant level head tank, the level of liquid in which is controlled by a weir of sufficient length.

**1.3.5 buoyancy correction :** The correction to be made to the readings of a weighing machine to take account of the difference between the upward thrust exerted by the atmosphere, on the liquid being weighed and on the reference weights used during the calibration of the weighing machine.

### 1.4 Units

The units used in this International Standard are the SI units, metre, kilogram, and second; the degree Celsius is used for convenience instead of the kelvin.

## 1.5 Notation

Symbol	Designation	Dimension	SI Units
$q_m$	Mass flow-rate	$MT^{-1}$	kg/s
$q_V$	Volume flow-rate	$L^3T^{-1}$	$m^3/s$
$m$	Mass	M	kg
$V$	Volume	$L^3$	$m^3$
$t$	Time	T	s
$\rho$	Density of liquid	$ML^{-3}$	$kg/m^3$
$\rho_a$	Density of air (at 20 °C and 1 bar*)	$ML^{-3}$	$kg/m^3$
$\rho_p$	Density of standard weights	$ML^{-3}$	$kg/m^3$
$s_x$	Estimated standard deviation		
$\sigma_x$	Standard deviation of variable $x$		
$e$	Uncertainty of measurement		
$e_s$	Systematic uncertainty		
$E_s$	Percentage systematic uncertainty		
$e_R$	Random uncertainty		
$E_R$	Percentage random uncertainty		

\* 1 bar =  $10^5$  Pa

## 1.6 Certification

If the installations for flow-rate measurement by the weighing method are used for purposes of legal metrology, they should be certified and registered by the national metrology service. Such installations are then subject to periodical inspection at

stated intervals. If a national metrology service does not exist, a certified record of the basic measurement standards (weight and time), and error analysis in accordance with this International Standard and ISO 5168, shall also constitute certification for legal metrology purposes.

## 2 Principle

### 2.1 Statement of the principle

#### 2.1.1 Static weighing

The principle of the flow-rate measurement method by static weighing (for schematic diagrams of typical installations, see figures 1A, 1B, 1C) is :

- to determine the initial mass of the tank plus any residual liquid;
- to divert the flow into the weighing tank (until it is considered to contain a sufficient quantity to attain the desired accuracy) by operation of the diverter, which actuates a timer to measure the filling time;
- to determine the final mass of the tank plus the liquid collected in it.

The flow-rate is then derived from the mass collected, the collection time and other data as discussed in clause 5 and annex A.

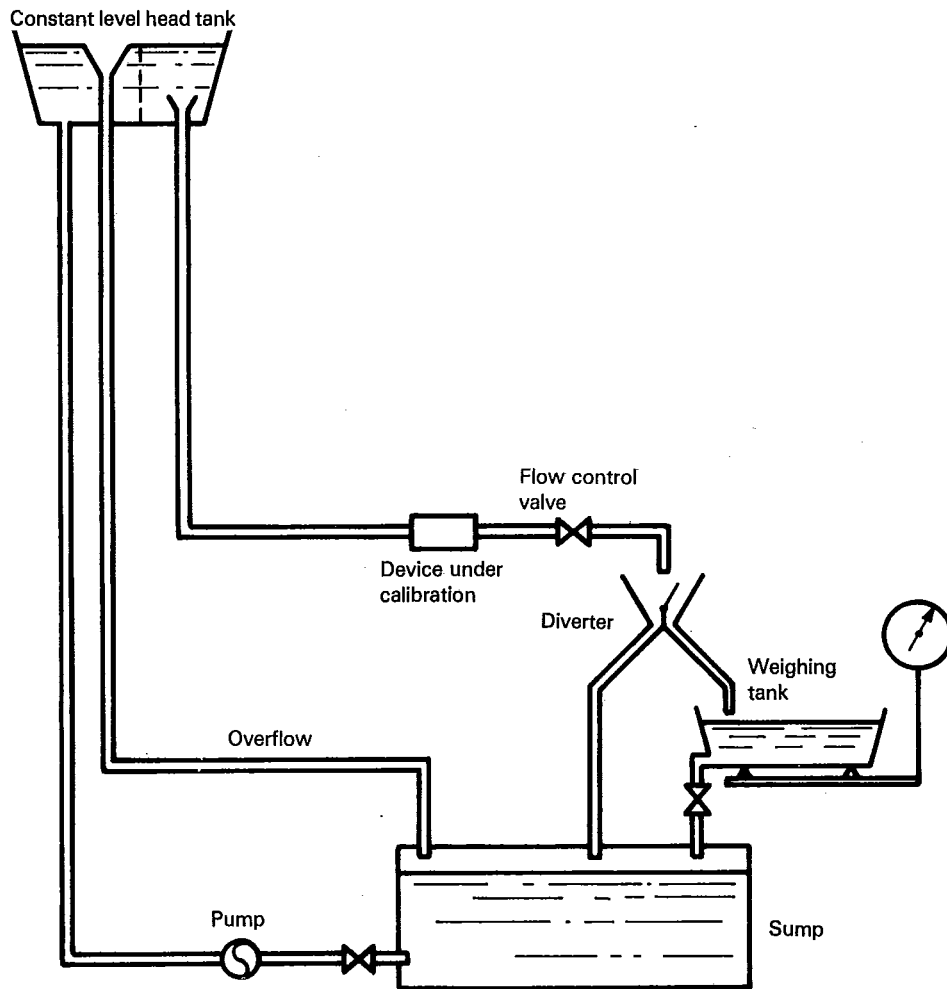


Figure 1A — Diagram of an installation for calibration by weighing (static method, supply by a constant level head tank)



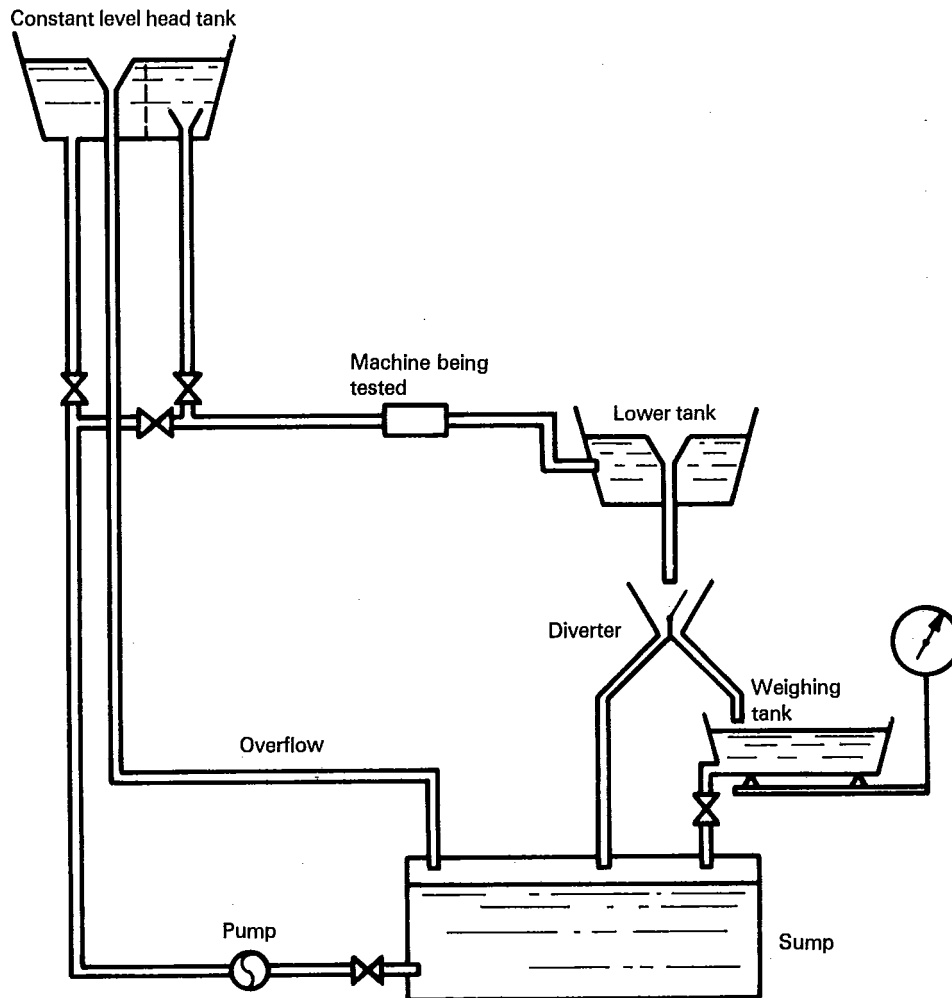


Figure 1B — Diagram of an installation for flow-rate measure by weighing (used for an hydraulic machine test; static method, supply by a constant level head tank)

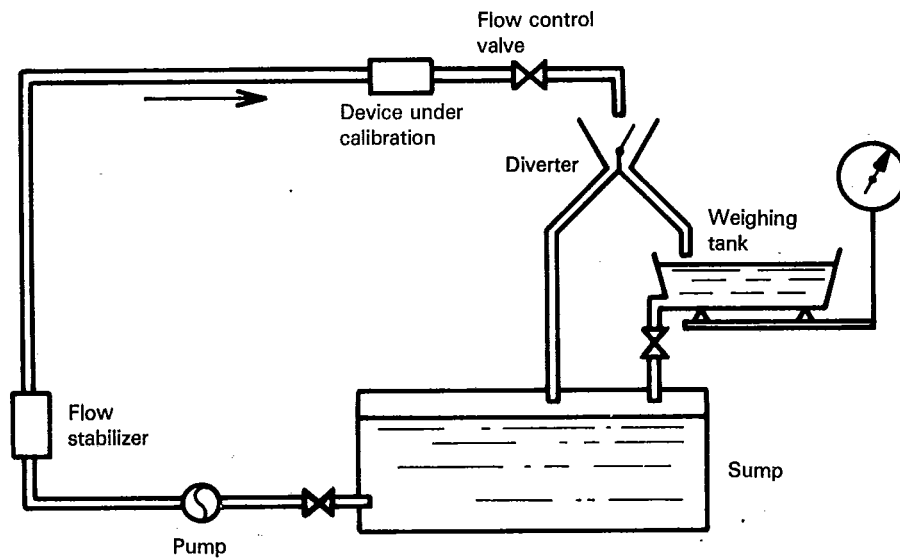


Figure 1C — Diagram of an installation for calibration by weighing (static method, direct pumping supply)

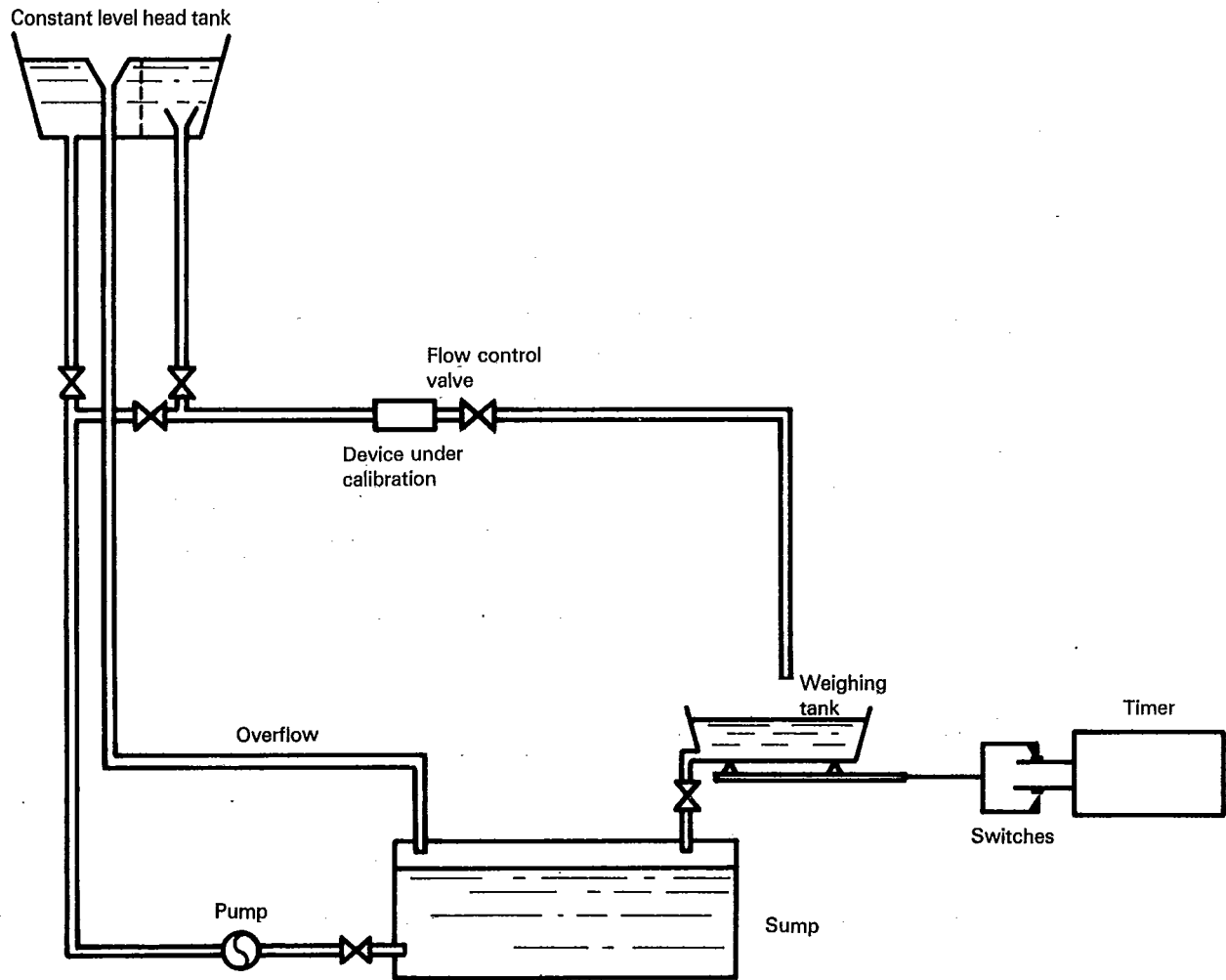


Figure 1D — Diagram of an installation for calibration by weighing (dynamic method, supply by a constant level head tank)

### 2.1.2 Dynamic weighing

The principle of the flow-rate measurement method by dynamic weighing (see figure 1D for a schematic diagram of a typical installation) is :

- to let the liquid collect in the tank to a predetermined initial mass, when the timer is then started;
- to stop the timer when a predetermined final mass of collected liquid is reached.

The flow-rate is then derived from the mass collected, the collection time and other data as discussed in clause 5 and annex A.

### 2.1.3 Comparison of instantaneous and mean flow-rate

It should, however, be emphasized that only the mean value of flow-rate for the filling is given by the weighing method. Instantaneous values of flow-rate as obtained on another instrument or meter in the flow circuit can be compared with the mean rate only if the flow is maintained stable during the measurement interval by a flow-stabilizing system, or if the instantaneous values are properly time-averaged during the whole filling period.

## 2.2 Accuracy of the method

### 2.2.1 Overall uncertainty on the weighing measurement

The weighing method gives an absolute measurement of flow which in principle requires only mass and time measurements. Provided that the precautions listed in 2.2.2 are taken, this method may be considered as one of the most accurate of all flow-rate measuring methods, and for this reason it is often used as a calibration method. When the installation is carefully constructed, maintained and used, an uncertainty of  $\pm 0,1\%$  (with 95 % confidence limits for the random part of that uncertainty) can be achieved.

### 2.2.2 Requirements for accurate measurements

The weighing method gives an accurate measurement of flow rate provided that :

- a) there is no leak in the flow circuit and there is no unmetered leakage flow across the diverter;

b) there is no accumulation (or depletion) of liquid in a part of the circuit by thermal contraction (or expansion) and there is no accumulation (or depletion) by change of vapour or gas volume contained unknowingly in the flow circuit;

c) necessary corrections for the influence of atmospheric buoyancy are made; this correction may be made when calibrating the weighing apparatus;

d) the weighing machine, the timer and means for starting and stopping it achieve the necessary accuracy;

e) the time required by the diverter for traversing is small with respect to the filling time, the timer being started and stopped while the diverter is crossing the hydraulic centre line;

f) in the case of the dynamic weighing method the effects of the dynamic phenomena are sufficiently small.

## 3 Apparatus

### 3.1 Diverter

The diverter is a moving device used to direct flow alternately along its normal course or towards the weighing tank. It can be made up of a conduit or moving gutter, or, better, by a baffle plate pivoting around a horizontal or vertical axis (see figure 2).

The motion of the diverter should be sufficiently fast (less than 0,1 s, for example) to reduce the possibility of a significant error occurring in the measurement of the filling time. This is accomplished by rapid diverter travel through a thin liquid sheet as formed by a nozzle slot. Generally, this liquid sheet has a length 15 to 50 times its width in the direction of diverter travel. The pressure drop across the nozzle slot should not exceed about 20 000 Pa to avoid splashing, air entrainment<sup>1)</sup> and flow across the diverter and turbulence in the weighing tank. This motion of the diverter can be generated by various electrical or mechanical devices, for example by a spring or torsion bar or by electrical or pneumatic actuators. The diverter should in no way influence the flow in the circuit during any phase of the measuring procedure.

1) In certain designs of nozzle slot, however, special vents to allow air ingress to the fluid jet may be necessary to ensure stable flow within the test circuit.

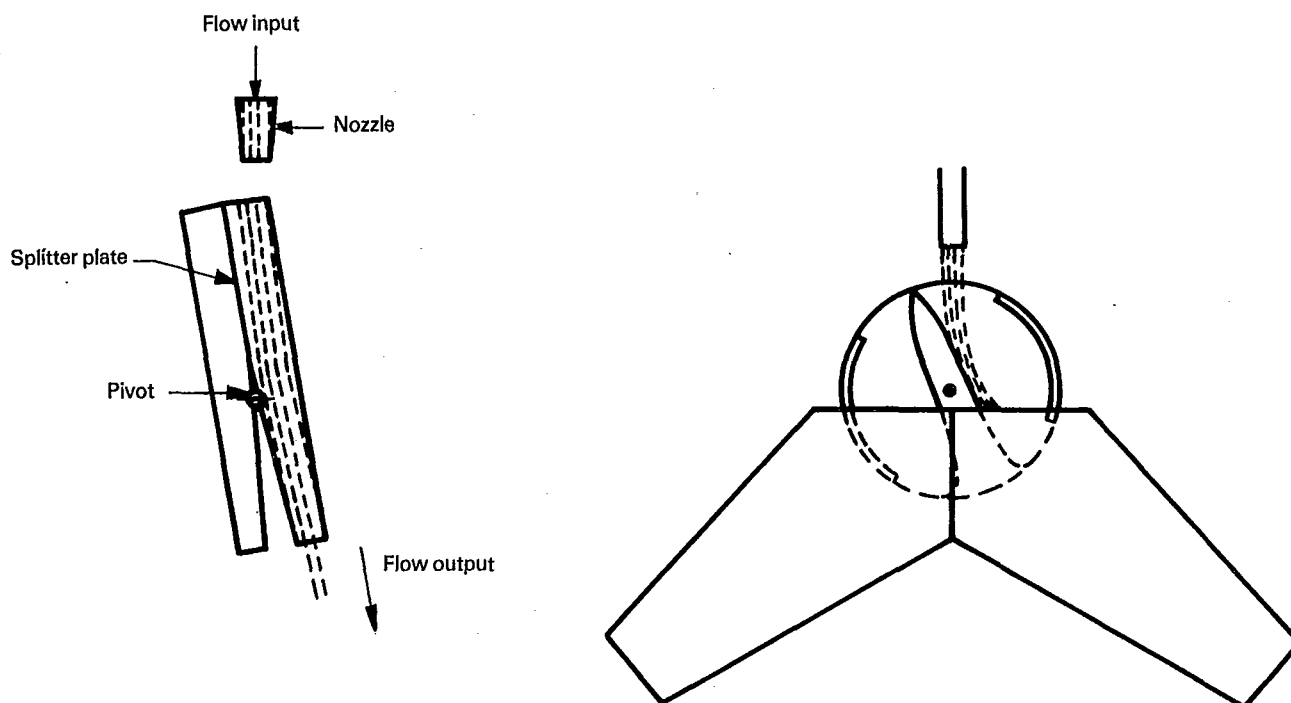


Figure 2 — Examples of diverter design

For large flow-rates which could involve excessive stresses, however, a diverter with a proportionately slow performance rate (1 to 2 s, for example) can be used provided that the operating law is constant and the variation of the flow-rate distribution as a function of the diverter stroke is preferably linear and is in any case known and can be verified.

Care shall be taken when designing the mechanical parts of the device and the diverter, as well as during frequent checks in service, that no leak or splash of liquid occurs either towards the outside or from one diverter channel to the other.

Besides a thin flat liquid stream, other shapes of liquid stream are permissible in the diverter duct, if the necessary corrections for the diverting time are applied as indicated in annex A.

### 3.2 Time-measuring apparatus

The time of discharge into the weighing tank is normally measured by an electronic counter with an in-built accurate time reference, for example a quartz crystal. The diversion period can thus be read to 0,01 s or better. The error arising from this source can be regarded as negligible provided that the discrimination of the timer display is sufficiently high and the equipment is checked periodically against a national time standard — for example, the frequency signals transmitted by certain radio stations.

The timer shall be actuated by the motion of the diverter itself through a switch fitted on the diverter (for example, optical or magnetic). Strictly speaking, the time measurement shall be started (or stopped) at the instant when the hatched areas in

figure 3, which represent flow variation with time, are equal. In practice, however, it is generally accepted that this point corresponds to the mid-travel position of the diverter in the fluid jet. The error will be negligible provided that the time of passage of the diverter through the stream is negligible in comparison with the period of diversion to the tank.

If the operating law of the diverter, if any, is identical in both directions (see figure 4), the timer may be started and stopped at the instant when the motion of the diverter is started in each direction; this is the case particularly when the time-flow rate law is linear.

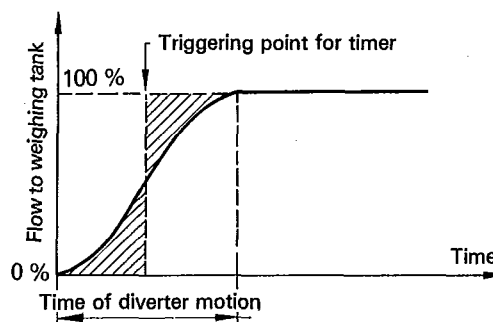
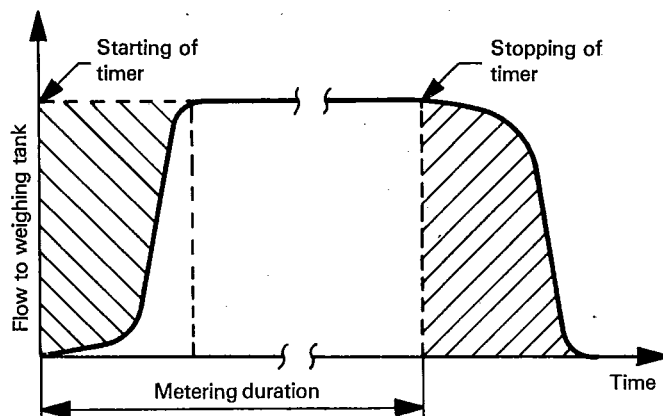


Figure 3 — Operational law of diverter

If, however, the error in the filling time measurement arising from the operation of the diverter and starting and stopping of the timer is not negligible, a correction should be made in accordance with the directions of annex A.



**Figure 4 — Time metering for a diverter the operation law of which is identical in both directions**

### 3.3 Weighing tank

The tank into which flow discharges during each measuring stage shall be of sufficient capacity so that the error in timing is negligible. Taking account of what is stated in 3.1 and 3.2, the filling time for the highest expected flow-rate shall be at least 30 s. Nevertheless, this time may be reduced provided that it is possible to determine experimentally, according to procedures such as described in annex A, that the required accuracy is achieved.

The tank may be of any shape but it is essential that it is perfectly leak-tight, and care should be taken to avoid liquid spillage. Internal walls or baffles may be required to reduce oscillations of the liquid in the tank and to improve structural rigidity.

The tank may be suspended from the weighing machine or may constitute the platform of the latter or may be placed on one of the platforms. To prevent sudden overloads detrimental to the weighing apparatus, it may be necessary to lock the tank in position on the scale during filling.

The tank may be drained by different means :

- by a gate-valve at the base, the leak-tightness of which shall be capable of being verified (free discharge, transparent hose, or leak detection circuit);
- or by a siphon fitted with an efficient and checkable siphon break;
- or by a self-priming or submersible pump.

The rate of draining shall be sufficiently high that test runs can follow each other at short intervals.

In all cases it shall be carefully checked that no pipe connections or electric wire links exist likely to transmit stresses between the weighing tank and the fixed parts of the installation;

indispensable links shall therefore be extremely flexible, and their flexibility verified during the calibration of the weighing machine.

### 3.4 Weighing machine

The weighing machine may be of any type — mechanical or with strain-gauge load cells, for example — provided that it offers the required sensitivity, accuracy and reliability. When the weighing method of measuring flow-rate is applied for the purposes of legal metrology, it is advisable to employ the weighing machine according to OIML Recommendations Nos. 3 and 28.

After its installation in the test facility, the weighing machine shall be calibrated over the whole measuring range using standard weights. Here it is advisable to follow OIML Recommendations Nos. 1, 2, 20 and 33.

The weighing machine shall be regularly maintained and its calibration shall be periodically checked. If the weights available are not sufficient in number or size to cover the whole measuring range, a calibration shall be made in steps by replacing the weights by liquid and by using standard weights to verify intervals accurately.

It should be noted that in view of the difference in buoyancy when calibrating the weighing machine with weights and when weighing an equivalent mass of liquid, a correction to the readings is necessary (see the calculation in 5.1).

### 3.5 Auxiliary measurements

To obtain the volume flow-rate from mass measurement, it is essential to know the density of the liquid with the required accuracy at the time of weighing.

If the liquid to be measured is reasonably pure and clean, it is acceptable to measure its temperature and to derive its density from a table of physical properties (see annex B for the case of water). Temperature may be measured with a simple mercury-in-glass thermometer or, better, by any device such as a resistance probe or thermocouple, preferably placed in the flow circuit where it is required to know the volume flow-rate. For the case of water, taking account of the small variation of density with temperature about ambient temperature, an accuracy of 0,5 °C is enough to ensure less than  $10^{-4}$  error on density evaluation.

If, however, the purity of the liquid is in doubt, it is essential to measure its density. To this end, a sample can be collected and its density measured either by a direct method, by weighing in a graduated cylinder on an analytical balance, or by an indirect method, for example by measuring the hydrostatic thrust exerted on a calibrated float (hydrostatic balance). Whatever the method used, the liquid temperature must be measured when measuring the density; in many cases it may be assumed that the relative variation of density with respect to temperature is the same as for the pure liquid.

## 4 Procedure

### 4.1 Static weighing method

In order to eliminate the effect of residual liquid likely to have remained in the bottom of the tank or adhering to the walls, a sufficient quantity of liquid shall first be discharged into the tank (or left at the end of draining after the preceding measurement) to reach the operational threshold of the weighing machine. This initial mass  $m_0$  will be recorded while the diverter directs the flow to storage, and while the flow-rate is being stabilized. After steady flow has been achieved, the diverter is operated to direct the liquid into the weighing tank, this operation automatically starting the timer. After collection of an appropriate quantity of liquid, the diverter is operated in the opposite direction to return the liquid to storage, automatically stopping the timer and thus allowing the filling time  $t$  to be determined. When the oscillations in the tank have subsided, the apparent final mass  $m_1$  of the weighing tank is recorded. The tank shall then be drained.

### 4.2 Dynamic weighing method

After steady flow has been achieved, the drain valve of the weighing tank is closed; as the mass of liquid in the tank increases, it overcomes the resistance due to counterpoise mass  $M_1$  on the end of the balance beam, which then rises and starts the timer. An additional mass  $\Delta m$  is then added to the pan of the balance beam to depress the latter. When the balance beam rises again, it stops the timer, and the filling time  $t$  is recorded. Mass  $\Delta m$  is used as  $(m_1 - m_0)$  in the subsequent calculation of the flow-rate.

There exist other possible methods of measurement; for example, automatic reading of the weighing machine indication.

### 4.3 Common provisions

It is recommended that at least two measurements be carried out for each of a series of flow-rate measurements if a subsequent analysis of random uncertainties is to be carried out.

The various quantities to be measured may be noted manually by an operator or be transmitted by an automatic data acquisition system to be recorded in numerical form on a printer or provide direct entry into a computer.

## 5 Calculation of flow-rate

### 5.1 Calculation of mass flow-rate

The mean mass flow-rate during the filling time is obtained by dividing the real mass  $m$  of the liquid collected by the filling time  $t$ :

$$q_m = \frac{m}{t} = \frac{m_1 - m_0}{t} \times \frac{1 - \frac{\rho_a}{\rho_p}}{1 - \frac{\rho_a}{\rho}}$$

If necessary,  $t$  is corrected in concordance with one of the procedures described in annex A to take into account the diverter timing error or the dynamic weighing timing error. The final term in this equation is a correction term introduced to take into account the difference in buoyancy exerted by the atmosphere on a given mass of liquid and on the equivalent mass in the form of weights made, for example, of cast iron, used when calibrating the weighing machine.

NOTE — In view of the relative magnitudes of the quantities, this equation can be written as follows with satisfactory approximation:

$$q_m \approx \frac{m_1 - m_0}{t} (1 + \epsilon)$$

where

$$\epsilon = \rho_a \left( \frac{1}{\rho} - \frac{1}{\rho_p} \right)$$

In the case where the liquid is water, it is sufficient to calculate the correction factor  $\epsilon$  from mean approximate values:

$$\rho = 1\,000 \text{ kg/m}^3$$

$$\rho_a = 1,21 \text{ kg/m}^3 \text{ (at } 20^\circ\text{C and 1 bar)}$$

$$\rho_p = 8\,000 \text{ kg/m}^3 \text{ (conventional mean value according to OIML Recommendation No. 33)}$$

Hence,

$$\epsilon = 1,06 \times 10^{-3}$$

and

$$q_m = 1,001\,06 \frac{m_1 - m_0}{t}$$

## 5.2 Calculation of volume flow-rate

The volume flow-rate is calculated from the mass flow-rate as computed in 5.1, and from the density of the liquid at the temperature of operation, as read from standard tables — for example, as given in annex B for water in the range of ambient temperatures. (In exceptional cases, it may be necessary to measure the density directly.)

$$q_V = \frac{q_m}{\rho} = \frac{m_1 - m_0}{\rho t} (1 + \epsilon)$$

## 6 Calculation of the overall uncertainty of the measurement of the flow-rate

The calculation of the uncertainty in the measurement of flow-rate should be carried out in accordance with ISO 5168 but for convenience the main procedures to be followed are given here as they apply to the measurement of flow-rate by the weighing method.

### 6.1 Presentation of results

Equation (3) of annex C should preferably be evaluated separately for the uncertainties due to the random and systematic components of error. Denoting the contributions to the uncertainty in the flow-rate measurement from these two

sources by  $(e_R)_{95}$  and  $e_s$  respectively when expressed in absolute terms, and by  $(E_R)_{95}$  and  $E_s$  when expressed as a percentage, the flow-rate measurement shall then be presented in one of the following forms :

a) Flow-rate =  $q$

$$(e_R)_{95} = \pm \delta q_1; \quad e_s = \pm \delta q_2$$

Uncertainties calculated according to ISO 5168.

b) Flow-rate =  $q$

$$(E_R)_{95} = \pm \delta q_3 \% ; \quad E_s = \pm \delta q_4 \%$$

Uncertainties calculated according to ISO 5168.

An alternative, although less satisfactory, method is to combine the uncertainties arising from random and systematic errors by the root-sum-square method. Even then, however, it is necessary to evaluate equation (3) for the random components since the value of  $(e_R)_{95}$  or  $(E_R)_{95}$  must be given. In this case, the flow-rate measurement shall be presented in one of the following forms :

c) Flow-rate =  $q \pm \delta q$

$$(e_R)_{95} = \pm \delta q_1$$

Uncertainties calculated according to ISO 5168.

d) Flow-rate =  $q(1 \pm 10^{-2} \delta q')$

$$(E_R)_{95} = \pm \delta q_3 \%$$

Uncertainties calculated according to ISO 5168.

## 6.2 Sources of error

Only the principal sources of systematic and random errors are considered below, the numerical values of errors mentioned being given as examples.

The sources of systematic and random errors are considered separately here, but it should be noted that only a single determination of flow-rate is being considered. It should also be noted that the purpose of the measurement is considered to be the determination of the mean flow-rate over the period of the diversion. Thus the effect of instability in the flow need not be considered provided that it is not so severe as to affect the operation of the diverter system.

### 6.2.1 Systematic errors

#### 6.2.1.1 Errors due to weighing machine

The systematic errors which may be associated with the use of a weighing machine may arise, for example in the case of a steelyard, from :

- the notch positions on the steelyard;
- evaluation of  $\epsilon$ .

Each notch position on the steelyard will be in error by an amount which ideally should be less than the discrimination of the weighing machine. In many cases, however, this ideal will not be attained, and a calibration of the weighing machine will produce an error distribution such as that shown in figure 5.

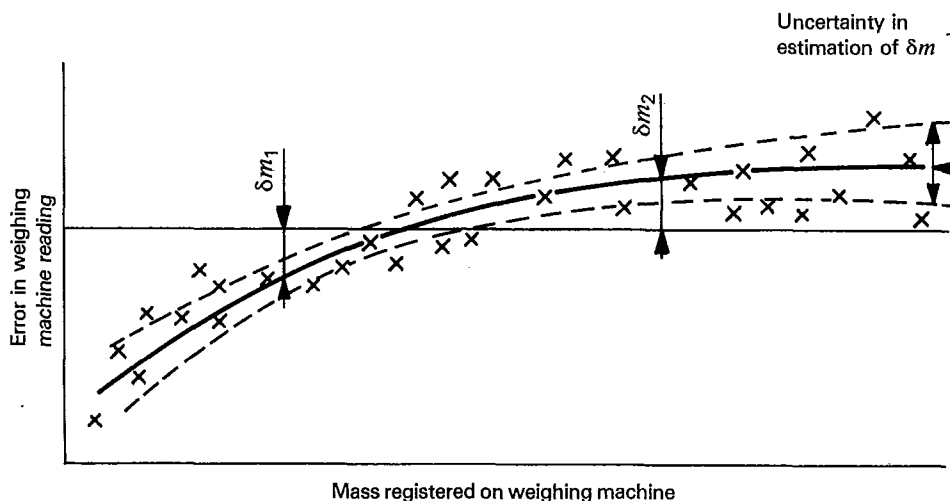


Figure 5 — Example of error distribution in calibration of weighing machine



In the general case, the best-fit curve through the individual points can be expressed as a polynomial :

$$\delta m = a_0 + a_1 m + a_2 m^2 + \dots + a_n m^n$$

It is recommended that the lowest order polynomial for the data should be chosen.

The systematic error in a determination of mass in the weighing tank,  $\delta(\Delta m)$ , is then given by :

$$\delta(\Delta m) = \delta m_2 - \delta m_1$$

In order to assess the value of this systematic error, it is therefore necessary to use a calibration curve of the form given in figure 5, but even after correcting mass differences by the appropriate amount there will be a residual uncertainty  $(e_s)_b$ , equal to the uncertainty in the determination of  $\delta(\Delta m)$ , introduced to the flow-rate measurement. This will be the uncertainty of the determination of the best curve through the individual calibration points.

The maximum permissible value of  $(E_s)_b$  shall be  $\pm 0,05$  % of the mass registered on the weighing machine. For a given absolute value of the uncertainty in the determination of  $\Delta(\delta m)$ , it will therefore be necessary to set a lower limit to the mass of water collected during a diversion in order to ensure that the uncertainty associated with this systematic error is always less than  $\pm 0,05$  %.

The correction for buoyancy,  $\epsilon$ , is determined from a knowledge of  $\rho$ ,  $\rho_a$  and  $\rho_p$ . There will be a systematic error arising from the value used, especially if standard values are taken as recommended in 5.1, but the magnitudes of the quantities involved are such that this error may be neglected, since it has an effect of less than 0,01 % on the flow-rate measurement.

#### 6.2.1.2 Errors due to timing device

Any error of the calibration of the timing device will result in a systematic error in the time measured for a diversion, but with modern equipment this will be negligible (less than 1 ms).

It is important that the discrimination of the timing device be adequate. Instruments with a digital display will give a reading which is in error by up to one last-order digit, the sign of the error depending on whether the digit is advanced at the end or the beginning of the corresponding time interval. In order to render this error negligible, the discrimination of any timing device used should be set to less than 0,01 % of the diversion time.

#### 6.2.1.3 Errors due to diverter system

Provided either that a correction is made for the timing error as described in annex A or that the triggering of the timing system is adjusted so that the timing error is zero, the uncertainty introduced to the measurement of flow-rate from this source will be equal to the uncertainty in the measurement of the timing error.

This uncertainty  $(e_s)_p$  may be calculated from the equation in annex A, clause A.1, using the general principle outlined in equation 3 of annex C, or from the uncertainty of the slope of the line in the graph in annex A (figure 7) when the alternative method 2 is used.

The value  $(E_s)_p$  must be less than 0,05 %.

#### 6.2.1.4 Errors due to density measurement

When the volumetric flow-rate has to be calculated, there will be a systematic error associated with the value used for the density of the liquid, which will arise from

- a) the measurement of the temperature of liquid in the installation;
- b) the use of the density measuring equipment or density tables.

As noted in 3.5, errors in the measurement of density in the case of water at ambient temperature will be insignificant provided that the temperature is measured to within  $\pm 0,5$  °C. This accuracy is easily attainable with simple thermometers, but it is important to ensure that the liquid flowing into the weighing tank is at constant temperature so that there is no possibility of the temperature of the liquid close to the thermometer being unrepresentative of that of the liquid in the tank as a whole.

When density tables are used, no significant error should be introduced, but if the density of a liquid is to be measured directly, an evaluation of the method used must be carried out in order to determine the uncertainty  $(e_s)_d$ , in the result. This value of  $(e_s)_d$  is then the value to be used in calculating the uncertainty of the volumetric flow-rate measurement.

Where volume flow-rates are to be measured and the liquid density is obtained by direct measurement, the method used shall be such as to ensure that the value of  $(E_s)_d$  is less than 0,05 %.

#### 6.2.2 Random errors

##### 6.2.2.1 Errors due to weighing machine

From a graph such as that shown in figure 5, the standard deviation of the distribution of points about the best-fit curve should be calculated and the 95 % confidence limits of the distribution determined using Student's *t*-table (see annex D). This value of confidence limits should then be multiplied by  $\sqrt{2}$  (since the determination of the mass of liquid collected during a diversion is obtained from the difference between two weighings) and the result,  $(e_R)_b$ , is the uncertainty due to random errors in the weighing machine.

The uncertainty due to random errors in the weighing machine,  $(E_R)_b$ , shall be less than  $\pm 0,1$  %; the minimum liquid mass to be weighed is selected according to this criterion.

### 6.2.2.2 Errors due to diverter system

The repeatability with which the duration of a diversion is measured depends on the repeatability of the movement of the diverter which triggers the timing device and on the accuracy with which the triggering position is set. For any given installation, this may be determined experimentally by setting the flow-rate to a steady value and then carrying out a series of, say 10 diversions for a fixed diversion period to provide a series of 10 estimations of the flow-rate.

This is repeated for several different diversion periods and, from the standard deviation  $s$  of each series of measurements, the 95 % confidence limits, i.e.  $\pm t_{95}s$  (see annex D) may be evaluated. Thus a graph of the form shown in figure 6 can be constructed for a well designed diversion system. It should be noted that the flow rate should be held steady or, preferably, normalized, for example, by using a reference flow-meter in the

circuit, during each set of measurements. Above some minimum diversion period, the 95 % confidence limits will be relatively constant, and the value so obtained should be used as the uncertainty,  $(E_R)_p$ , in the flow-rate measurement due to random effects in the diverter system.

It should be noted that  $(E_R)_p$  includes the scatter resulting from the readings of the scale of the weighing machine.

It is important that  $(E_R)_p$  be evaluated at several flow-rates over the range of the system since its value can be flow-rate-dependent.

The uncertainty due to random errors from this source,  $(E_R)_p$ , shall be less than 0,1 %. Attaining these limits will require the use of some minimum diversion period, which will have to be determined for a given installation from a knowledge of the absolute values of these uncertainties.

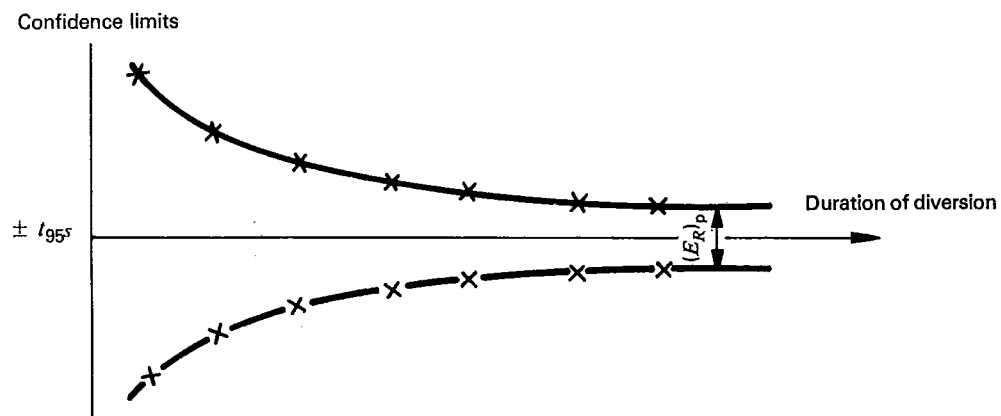


Figure 6 — Typical graph used in evaluation of  $(E_R)_p$  for a diverter system

### 6.3 Calculation of uncertainty in flow-rate measurement

#### 6.3.1 General

The uncertainty associated with a measurement of flow-rate is obtained by combining the uncertainties arising from the sources described in 6.2. Although "systematic" errors have been distinguished from "random" errors, the probability distribution of the possible values of each systematic component is essentially Gaussian, and, in accordance with ISO 5168, the combination of all the uncertainties may therefore be made by the root-sum-square method.

Although all the uncertainties should be considered, only those set out in 6.2 need be included in the analysis if the measurements have been made in accordance with this International Standard since other sources of error will make a negligible contribution to the overall uncertainty.

Hence, the relative systematic uncertainty in a volume flow-rate measurement is given by

$$E_s = \pm 100 \sqrt{\left[\frac{(e_s)_b}{m}\right]^2 + \left[\frac{(e_s)_p}{t}\right]^2 + \left[\frac{(e_s)_d}{\rho}\right]^2 + \left[\frac{(e_s)_\varepsilon}{m}\right]^2 + \left[\frac{(e_s)_t}{t}\right]^2} \%$$

The uncertainties  $(e_s)_\varepsilon$  and  $(e_s)_t$  can be generally omitted.

And the relative random uncertainty at the 95 % confidence level is given by

$$(E_R)_{95} = \pm 100 \sqrt{\left[\frac{(e_R)_b}{m}\right]^2 + \left[\frac{(e_R)_p}{t}\right]^2 + \left[\frac{(e_R)_d}{\rho}\right]^2} \%$$

The uncertainty  $(e_R)_d$  can be generally omitted.

#### 6.3.2 Example of calculation

The example taken here is one in which a steelyard registered a mass of 20 000 kg of water collected over a measured period of 40,00 s, and where the volume flow-rate of the water is required. The value of the density obtained by measuring the temperature of the water in the weighing tank with a mercury-in-glass thermometer and the use of density bottles to measure a sample of the water, was 1 000,34 kg/m<sup>3</sup>.

The example considers only the sources of error listed in 6.2, and uses values of uncertainty for these sources of error which are typical of a high accuracy flow-measurement facility. It must be emphasized, however, that in any particular case the calculation must be carried out separately since other sources of error may exist and the values of uncertainty corresponding to any given source of error may vary.

#### 6.3.2.1 Systematic errors

It is assumed here that the procedures outlined in 6.2 have already been carried out in order to provide the values of systematic uncertainties which are used below.

The systematic uncertainty due to the weighing machine arose, in this example, from the notch positions and the buoyancy correction; these components, denoted by  $(e_s)_b$  and  $(e_s)_\varepsilon$  respectively, typically have values of  $\pm 0,05 \%$  and  $\pm 0,005 \%$ , corresponding to values of  $\pm 10$  kg and  $\pm 1$  kg in this particular example.

The systematic uncertainty due to the timing device  $(e_s)_t$ , is typically less than 0,001 s, and so this value will be used for the purpose of this example.

The systematic uncertainty due to the diverter system,  $(e_s)_p$ , is typically  $\pm 0,025$  s.

The systematic uncertainty in the measurement of density,  $(e_s)_d$ , is typically  $\pm 0,01 \%$ , corresponding to  $\pm 0,1$  kg/m<sup>3</sup> in this case.

#### 6.3.2.2 Random errors

The confidence limits of a curve such as that given in figure 5 are typically  $\pm 0,05 \%$ , and so the random uncertainty,  $(e_R)_b$ , in the difference between the two weighings is  $\pm 0,07 \%$ . Thus the random uncertainty due to the weighing machine corresponds to an uncertainty of  $\pm 14$  kg in the present example.

The random uncertainty due to the diverter system  $(e_R)_p$ , is typically  $\pm 0,01$  s.

The random uncertainty in the evaluation of density,  $(e_R)_d$ , is typically  $\pm 0,01 \%$ , corresponding here to  $\pm 0,1$  kg/m<sup>3</sup>.

#### 6.3.2.3 Calculation of uncertainty in flow-rate measurement

The percentage systematic uncertainty,  $E_s$ , in the flow-rate measurement is given by

$$E_s = \pm 100 \sqrt{\left[\frac{(e_s)_b}{m}\right]^2 + \left[\frac{(e_s)_\varepsilon}{m}\right]^2 + \left[\frac{(e_s)_t}{t}\right]^2 + \left[\frac{(e_s)_p}{t}\right]^2 + \left[\frac{(e_s)_d}{\rho}\right]^2} \%$$

Thus,

$$\begin{aligned} E_s &= \pm 100 \sqrt{\left(\frac{10}{20000}\right)^2 + \left(\frac{1}{20000}\right)^2 + \left(\frac{0,001}{40}\right)^2 + \left(\frac{0,025}{40}\right)^2 + \left(\frac{0,1}{1000,34}\right)^2} \\ &= \pm 100 \sqrt{0,654 \times 10^{-6}} \quad \% \\ &= \pm 0,08 \quad \% \end{aligned}$$

The percentage random uncertainty,  $(E_R)_{95}$ , in the flow-rate measurement is given by

$$(E_R)_{95} = \pm 100 \sqrt{\left[\frac{(e_R)_b}{m}\right]^2 + \left[\frac{(e_R)_p}{t}\right]^2 + \left[\frac{(e_R)_d}{\rho}\right]^2} \quad \%$$

Thus,

$$\begin{aligned} (E_R)_{95} &= \pm 100 \sqrt{\left(\frac{14}{20\,000}\right)^2 + \left(\frac{0,01}{40}\right)^2 + \left(\frac{0,1}{1\,000,34}\right)^2} \quad \% \\ &= \pm 100 \sqrt{0,562 \times 10^{-6}} \quad \% \\ &= \pm 0,075 \quad \% \end{aligned}$$

Thus the flow-rate measurement results may be presented as :

$$\text{Flow-rate} = 1,001\,06 \frac{m_1 - m_0}{\rho t} = 0,500\,4 \text{ m}^3/\text{s}$$

$$(E_R)_{95} = \pm 0,075 \quad \%$$

$$E_s = 0,08 \quad \%$$

Uncertainties calculated according to ISO 5168.

It will be noted that some of these uncertainties have been shown to be negligible, but they are included here to illustrate the calculation method.

## Annex A

### Corrections on the measurement of filling time

Experience has shown that, for a well-designed system, the error occurring due to switching the timer on and off for one start-stop cycle of the diverter may correspond to a value of 0 to 25 ms. This error is dependent upon the flow-rate, the velocity of traverse in each direction of the diverter tip through the liquid flow, and the exact location of the timer actuator with respect to the liquid flow emerging from the nozzle slot. This error should not be assumed to be insignificant, but should be evaluated by experimental tests, using the procedure described in the following clauses.

#### A.1 Static weighing method

##### A.1.1 Method 1

When steady flow is established at the flow control valve, a standard test is run to determine the flow-rate. Then a series of short flows or bursts of flow (as many as 25 bursts) are deflected into the weighing tank without resetting the timer or the scales; the flow is then determined from the totalized mass and totalized time. To complete the run, a second standard determination is made on the steady flow, and the two standard determinations are averaged. Results obtained are then compared with the totalized flow determination.

If the totalized mass for  $n$  bursts is about equal to that of the standard run, it can be shown that the average timing error  $\Delta t$  due to chronograph control for one cycle is closely equal to :

$$\Delta t = \frac{t}{n-1} \left\{ \frac{q}{q'} \times \frac{\sum_1^n \Delta m_i / \sum_1^n t_i}{(m_1 - m_0)/t} - 1 \right\}$$

where

$(m_1 - m_0)/t$  is the flow-rate determined by the standard procedure;

$\frac{\sum_1^n \Delta m_i}{\sum_1^n t_i}$  is the flow-rate determined from the totalized mass and totalized time for  $n$  bursts;

$q$  and  $q'$  are flow-rates during the standard run and during the  $n$  bursts respectively, as measured by a self-contained meter in the flow circuit; the corrective term  $q/q'$  takes into account the flow-rate variations, if any, between both measuring runs.

After this procedure has been repeated over a wide range of flow-rates, it will be possible, on any further measurement, to correct the measured filling time by the value  $\Delta t$  so determined.

##### A.1.2 Method 2

The following alternative method of setting the diverter timer actuator may also be employed.

The normal flow-rate control mechanism of the hydraulic circuit should first be set to give a flow-rate close to the maximum flow-rate capability of the system, with a good-quality flow-rate meter in the circuit. The system is run at this condition for several hours, during which many successive measurements of flow-rate are made using different diversion times. Suggested times are "normal", and 0,2, 0,1 and 0,05 of "normal". The highest number of tests will be required at the 0,05 of "normal" (or long), with the lowest number of tests at the "normal" diversion time. During each of these times the average reading on the flow-rate meter should be taken as accurately as possible.

The results obtained should be fitted into the following equation, in which  $\Delta t$  is the required timing error of the diverter system :

$$\Delta t \left( \frac{1}{t_{q_i}} - \frac{1}{t_{q_n}} \right) = \frac{(q_i - q_n) - (\bar{q}_{it} - \bar{q}_{nt})}{q_n}$$

where

$t_{q_i}$  is the diversion time for a particular "short" test;

$t_{q_n}$  is the diversion time for the "normal" length test occurring nearest in diurnal time in the testing sequence;

$q_i$  is the flow-rate calculated for the particular diversion time  $t_{q_i}$ ;

$q_n$  is the flow-rate calculated for the "normal" diversion time  $t_{q_n}$  occurring nearest in diurnal time in the testing sequence;

$\bar{q}_{it}$  is the average flow-rate meter reading during time  $t_{q_i}$ ;

$\bar{q}_{nt}$  is the average flow-rate meter reading during time  $t_{q_n}$ .

The values obtained for the right-hand side of this equation should be plotted against  $(1/t_{q_i} - 1/t_{q_n})$  as shown in figure 7. The points should define a straight line passing through the origin, and the slope of which is equal to  $\Delta t$ .

If a significant value of  $\Delta t$  is obtained, the diverter timer actuator should be adjusted to minimize the value of the error as shown by repeated testing.

The procedure should be repeated at a few lower flow-rates to examine whether or not the value of  $\Delta t$  obtained is significantly flow-rate-dependent. If significant changes in the  $\Delta t$  value are obtained, it will be necessary to improve the operation of the diverter system or to introduce a variable correction time  $\Delta t$  to be applied to the diversion time.

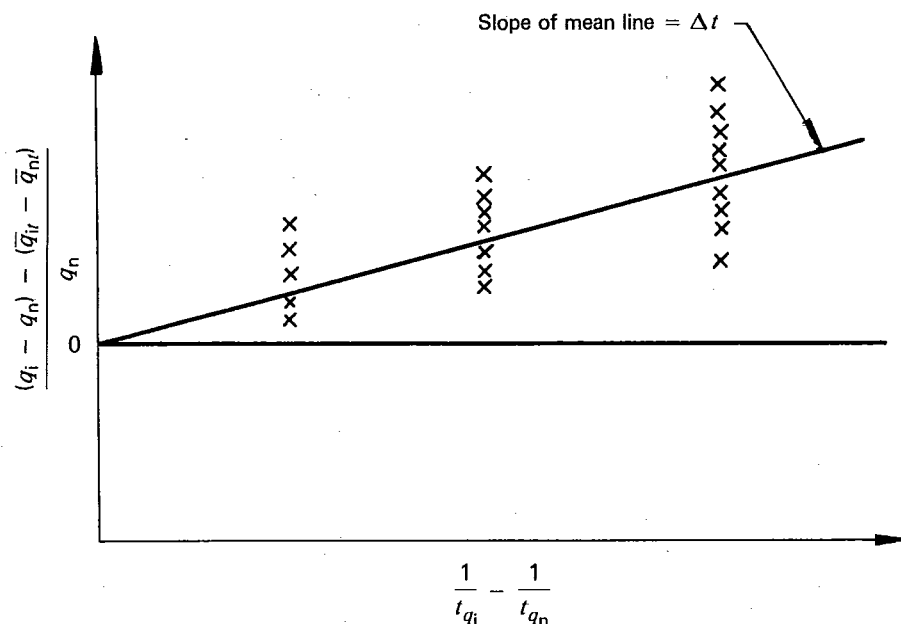


Figure 7 — Plotting of results of diverter timer actuator as given in A.1.2

## A.2 Dynamic weighing method

This procedure involves movement of the beam of the weighing machine just prior to both start and stop actuations of the timer.

Four important dynamic phenomena take place during the dynamic weighing cycle, namely :

- a change in the impact force of the falling liquid between the initial and final weighing points;
- collection of an extra amount of liquid from the falling column by the rising level in the tank;
- forces due to waves in the tank;
- a change in the inertia of the weighing machine and liquid in the weighing tank, with a resultant change of time required to accelerate the balance beam to the timer actuation point.

Generally, the decrease in impact force is equal and opposite to the additional weight of liquid collected, so that these two effects cancel each other.

Oscillations of liquid within the weighing tank may have a serious influence on the precision of the method. Devices prescribed in 3.3 can reduce, but not eliminate completely, this

undesirable phenomenon, which is always most pronounced at higher rates of flow.

Changes in inertia between the initial and final weighing points can affect indicated flow-rate by up to 0,5 % if the error  $\Delta t$  in measured time  $t$  is not accounted for. This error is approximately<sup>1)</sup>

$$\frac{\Delta t}{t} = \left[ \frac{6L\alpha}{g} \right]^{1/3} \left[ \frac{\Delta m}{t} \right]^{2/3} \frac{(M_1 + \Delta m)^{1/3} - M_1^{1/3}}{\Delta m}$$

where

$L\alpha$  is the distance travelled by the end of a balance beam of length  $L$  deflected through an angle  $\alpha$  from rest to the timing point;

$M_1$  ordinarily will include the masses of the weighing tank and initial liquid therein, and possibly other masses depending on the weighing machine used.

The corrected collection time in this case is  $(t - \Delta t)$ .

This error  $\Delta t$  can be reduced in conventional weighing applications by limiting the deflection  $\alpha$ . Alternatively, static weighing experiments can be compared with those using the dynamic technique to determine  $\Delta t$ ; the results can then be used to test the above equation for applicability and to evaluate the constants therein. On smaller dynamic-weighing systems, the inertia effect can be practically eliminated by using a substitution weighing technique.

1) SHAFER, M.R., and RUEGG, F.W. "Liquid flowmeter calibration techniques". Trans. ASME., Vol 80, No. 7, Oct. 1958.

## Annex B

### Density of pure water

Temperature °C	Density kg/m <sup>3</sup>
0	999,84
2	999,94
4	999,97
6	999,94
8	999,85
10	999,70
12	999,50
14	999,24
16	998,94
18	998,60
20	998,20
22	997,77
24	997,30
26	996,78
28	996,23
30	995,65
32	995,03
34	994,37

## Annex C

### Definition of terms and procedures used in error analysis

#### C.1 Definition of the error

The error in the estimate of a quantity is the difference between that estimate and the true value of the quantity.

No measurement of a physical quantity is free from uncertainties arising either from systematic errors or from the random dispersion of measurement results. Systematic errors cannot be reduced by repeating measurements since they arise from the characteristics of the measuring apparatus, the installation, and the flow characteristics. However, a reduction in the random error may be achieved by repetition of measurements, since the random error of the mean of  $n$  independent measurements is  $\sqrt{n}$  times smaller than the random error of an individual measurement.

#### C.2 Definition of uncertainty

The range within which the true value of a measured quantity can be expected to lie with a suitably high probability is termed the uncertainty of the measurement. For the purposes of this International Standard, the probability to be used shall be the 95 % level.

#### C.3 Definition of the standard deviation<sup>1)</sup>

If a variable  $X$  is measured several times, each measurement being independent of the others, then the standard deviation  $s_X$  of the distribution of  $n$  measurements  $X_i$  is :

$$s_X = \frac{\left[ \sum_{i=1}^n (X_i - \bar{X})^2 \right]^{1/2}}{n - 1} \quad \dots(1)$$

where

$\bar{X}$  is the arithmetic mean of the  $n$  measurements of the variable  $X$ ;

$X_i$  is the value obtained by the  $i$ th measurement of the variable  $X$ ;

$n$  is the total number of measurements of  $X$ .

For brevity,  $s_X$  is normally referred to as the standard deviation of  $X$ .

#### C.4 Assessment of uncertainty

##### C.4.1 Random errors

If the true standard deviation,  $\sigma_X$ , is known, the range  $\pm 1,96 \sigma_X$  would be expected to contain 95 % of the popula-

tion, i.e. there would be a probability of 0,05 of the interval  $\bar{X} \pm 1,96 \sigma_X$  not including the true value of  $X$ , and  $\pm 1,96 \sigma_X$  is the uncertainty of the measurement.

In practice, of course, it is possible to obtain only an estimate of the standard deviation since an infinite number of measurements would be required in order to determine it precisely, and the confidence limits must be based on this estimate. The "t distribution" for small samples should then be used to determine the uncertainty at the 95 % confidence level, as described in annex D.

##### C.4.2 Systematic errors

The procedure to be followed for arriving at the uncertainty associated with a systematic error depends on the information available on the error itself.

a) If the error has a unique, known value then this should be added to (or subtracted from) the result of the measurement, and the uncertainty in the measurement due to this source is then taken as zero.

b) When the sign of the error is known but its magnitude has to be estimated subjectively, the mean estimated error should be added to the result of the measurement (paying due observance to sign) and the uncertainty taken as one-half of the range within which the error is estimated to lie. This is illustrated in figure 8, where the measured value is denoted by  $M$  and the systematic error is estimated to lie between  $\delta t_1$  and  $\delta t_2$  [giving a mean estimated error of  $\frac{1}{2} (\delta t_1 + \delta t_2)$ ]. The result,  $R$ , to be used is then given by :

$$R = M + \frac{\delta t_1 + \delta t_2}{2}$$

with an uncertainty of

$$\pm \frac{\delta t_1 - \delta t_2}{2}$$

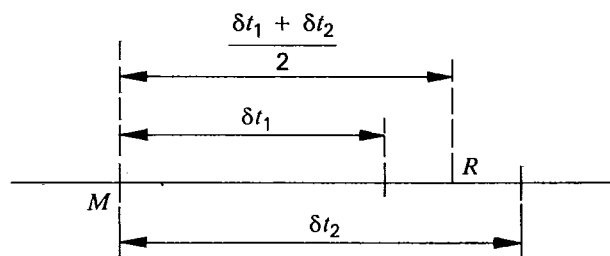


Figure 8 — Illustration of the correction to allow for mean estimated error

Putting the mean estimated error equal to the mean of the estimated maximum and minimum values assumes implicitly that the systematic error is regarded as asymmetric.

1) The standard deviation defined here is more accurately called "estimated standard deviation" by statisticians.



c) When the magnitude of the systematic uncertainty can be assessed experimentally, the uncertainty should be calculated as described in C.4.1 for random errors, with the measured value being adjusted as described above. Such a situation would arise where, for example, a weighing machine is calibrated and adjusted. Any given reading will have a systematic error, but individual readings will be distributed in a random manner about the true values; in applying a global uncertainty to the weighing-machine result, this random uncertainty can be used to set limits about the measured value.

d) When the sign of the error is unknown and its magnitude is assessed subjectively, the mean estimated error is equal to zero and the uncertainty should again be taken as one-half of the estimated range of the error. This is illustrated in figure 9, where the notation is as before. In this case,  $[\delta t_1] = [\delta t_2]$  so that the uncertainty is  $\pm \delta t$ .

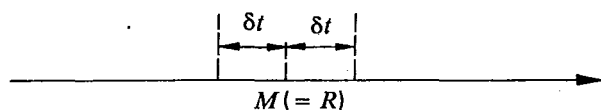


Figure 9 — Uncertainty =  $\pm \delta t$

## C.5 Propagation of errors

If the various independent variables, the knowledge of which allows computation of the flow-rate, are  $X_1, X_2, \dots, X_k$ , then the flow-rate  $q$  may be expressed as a certain function of these variables:

$$q = f(X_1, X_2, \dots, X_k) \quad \dots(2)$$

If the uncertainties associated with the variables  $X_1, X_2, \dots, X_k$  are  $e_1, e_2, \dots, e_k$ , then the uncertainty  $e_q$  of the flow-rate is defined as:

$$e_q = \left[ \left( \frac{\partial q}{\partial X_1} e_1 \right)^2 + \left( \frac{\partial q}{\partial X_2} e_2 \right)^2 + \dots + \left( \frac{\partial q}{\partial X_k} e_k \right)^2 \right]^{1/2} \quad \dots(3)$$

where  $\frac{\partial q}{\partial X_1}, \frac{\partial q}{\partial X_2}, \dots, \frac{\partial q}{\partial X_k}$  are partial derivatives. (See ISO 5168.)

The percentage uncertainty,  $E_q$ , is given by

$$E_q = 100 \frac{e_q}{q} \%$$

## Annex D

### Student's *t*-distribution

The uncertainty at the 95 % confidence level may be found as follows :

- 1) if  $n$  is the number of measurements,  $n - 1$  is taken as the number of degrees of freedom,  $\nu$ ;
- 2) obtain the value of  $t$  for the appropriate number of degrees of freedom,  $n - 1$ , from the table;
- 3) calculate the standard deviation,  $s_X$ , of the distribution of the measurements of the quantity  $X$ ;
- 4) the range of values within which any reading would be expected to lie with 95 % confidence is  $X \pm ts_X$ ;
- 5) the range of values within which the true mean would be expected to lie with 95 % confidence is  $\bar{X} \pm ts_X/\sqrt{n}$ .

Table — Values of Student's  $t$

Number of degrees of freedom $\nu = n - 1$	$t$ Confidence level 95 %
1	12,706
2	4,303
3	3,182
4	2,776
5	2,571
6	2,447
7	2,365
10	2,228
15	2,131
20	2,086
30	2,042
60	2,000
$\infty$	1,960